

Class: XII Probability

1. A bag contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random without replacement from the selected bag, find the probability of one of them being red and another black. Ans: $\frac{22}{45}$
2. An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained. Ans: 2, 1
3. 3 cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution. Ans: $\frac{3}{4}$
4. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X=4) = P(X=2)$. Find the probability of success. Ans: $\frac{1}{4}$
5. Three machines E1, E2 and E3 in a certain factory producing electric bulbs, produce 50%, 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E1 and E2 are defective and that 5% of those produced by machine E3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective. Ans: $\frac{17}{400}$
6. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained. Find mean and variance. Ans: $\frac{17}{3}$, $\frac{14}{9}$.
7. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B). Ans: $\frac{1}{5}$, $\frac{1}{6}$ or $\frac{5}{6}, \frac{4}{5}$
8. In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B. Ans: $\frac{11}{31}$.
9. Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and variance. Ans: mean = $\frac{2}{3}$, variance = $\frac{5}{9}$.

10. Assume that each born child is equally likely to be a boy or a girl. If a family has two children what is the conditional probability that both are girls ? given that (i) the youngest is a girl (ii) at least one is a girl. Ans: $1/2$, $1/3$
11. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials there will be at least 3 successes. Ans: $918\left(\frac{1}{4}\right)^5$.
12. 3 cards are drawn at random without replacement from a shuffled pack of 52 cards, find the probability distribution of number of red cards. Hence find the mean . Ans: $3/2$
13. A card from a pack of 52 playing is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.
Ans: $110/539$
14. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement . find the probability distribution of number of defective bulbs. Find the mean. Ans: $4/3$
15. There are 3 coins. One is a two headed coin (having head on both sides), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed and it shows head. What is the probability that it was the two headed coin? Ans: $20/47$
16. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean. Ans: $14/3$
17. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver? Ans: $7/52$
18. Five cards are drawn one by one with replacement from a well shuffled deck of 52 cards. Find the probability that (i) all the five cards are diamonds (ii) only 3 cards are diamonds (iii) none is a diamond Ans: $\left(\frac{1}{4}\right)^5$, $90\left(\left(\frac{1}{4}\right)\right)^5$, $\left(\frac{3}{4}\right)^5$
19. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A? Ans: $9/10$

20. P speaks truth in 70% of the cases, Q in 80% of the cases. In what percent of the cases are they likely to agree in stating the same fact? Do you think, when they agree, means both are speaking truth? Ans: 62%
21. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent., find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time. Ans: $\frac{26}{49}$.
22. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean. Ans: $\frac{6}{5}$.
23. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient. Ans: $\frac{14}{29}$
24. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee, asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner, if the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not. Ans: $\frac{5}{11}$, not fair.
25. Often it is taken that a truthful person commands more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Ans: $\frac{4}{9}$
26. How many times must a man toss a fair coin so that the probability of having at least one head is more than 80%? Ans: $n = 3$.
27. Two cards are drawn simultaneously (without replacement) from a well shuffled pack of 52 cards. Find the mean and variance. Ans: $\frac{25}{51}$.
28. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes and hence find its mean. Ans: $\frac{2}{3}$.
29. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen

at random from the college and he has 'A' grade, what is the probability that the student is a hostler? Ans: 9/13.

30. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black. Ans: 4/17
31. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1,2,3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2,3 or 4 with the die? Ans: 8/11
32. In a certain college, 4% of boys and 1% of girls are taller than 1.75 m. further more ,, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 m. find the probability that the selected student is girl. Ans: 3/11
33. Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem. Ans: 2/3, 1/2
34. A random variable X has the following probability distribution
- | | | | | | | | | |
|-------|---|---|----|----|----|-------|--------|------------|
| X: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P(x): | 0 | k | 2k | 2k | 3k | k^2 | $2k^2$ | $7k^2 + k$ |
- Determine (i) k (ii) $P(x < 3)$ (iii) $P(x > 6)$ (iii) $P(0 < x < 3)$. Ans: 1/10, 3/10, 17/100, 3/10.
35. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it actually a six. Ans: 3/8
36. Suppose 5% of men and 0.25 % of women have grey their. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females. Ans: 20/21
37. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold? Ans: 2/3
38. A factory has two machines A and B. past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? Ans: $\frac{1}{4}$

39. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II at random. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. Ans: $\frac{16}{31}$
40. There are 3 coins. One is a two headed coin (having heads on both faces), another is a biased coin that comes up heads 75% of the times and third is an unbiased coin. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two-headed coin? Ans: $\frac{4}{9}$.
41. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing? Ans: $\frac{11}{243}$
42. There are 2 bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bag and it is found to be white. Find the probability that it was drawn from bag I. Ans: $\frac{40}{61}$.
43. An experiment succeeds twice as often as it fails. Find the probability that in the next 6 trials, there will be at least 4 successes. Ans: $\frac{31}{9} \left(\frac{2}{3}\right)^4$
44. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of club. Ans: $\frac{11}{50}$.
45. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs. Ans: $\frac{7}{15}, \frac{7}{15}, \frac{1}{15}$
46. A bag contains 4 balls . two balls are drawn at random and are found to be white. What is the probability that all balls are white? Ans: $\frac{3}{5}$.
47. An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of 3 balls. Find the mean and variance. Ans: $\frac{9}{7}, \frac{24}{49}$.
48. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer, given that he answered it correctly? Ans: $\frac{9}{11}$
49. A die is thrown again and again until 3 sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die. Ans: $\frac{625}{3 \times 6^5}$

50. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target, what is the probability that (i) the target is hit? (ii) exactly one of them hits the target? Ans: $\frac{3}{5}$, $\frac{7}{15}$.

51. Three bags contain balls as shown in the table below:

| Bag | white balls | black balls | red balls |
|-----|-------------|-------------|-----------|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 3 | 2 |

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?

Ans: $\frac{5}{17}$.

52. Two groups are competing for the positions on Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and corresponding probability is 0.3 if second group wins. Find the probability that the new product introduced was by second group.

Ans: $\frac{2}{9}$

53. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4. Ans: $\frac{3}{7}$

54. From a lot of 30 bulbs, which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the mean and variance of the number of defective bulbs. Ans: $\frac{16}{25}$.